Simple Mathematical Models are Insufficient in Explaining Vertical Jumping

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SUMMARY

We think of agility as the ability to move at fast speeds while executing motor control strategies that redirect body motion and reposition limbs [1]. Jumping to high heights and to long distances correlates with other aspects of agility, such as maximum sprint speeds [2]. To gain intuition on how the legs might behave as an actuator limited by mechanical characteristics, we test models of varying physiological complexity and compare model predicted ground reaction forces to measured data. We seek one model that predicts jumping behavior independent of jump depth. Through our approach we learn of the insufficiencies of simple models.

INTRODUCTION

Maximum height vertical jumping presents a useful entry point for studying agility. When executing a vertical jump, we model the legs working as a mechanical actuator that pushes against the ground to accelerate the body. Our research seeks to characterize the mechanical limits of the actuator to understand how higher jumps could be enabled and what limits performance. We use a modeling method that builds from the simplest morphological and physiological system—a point mass body with a massless un-segmented leg that has no force dependence on actuator length, velocity, or activation—to more complex systems with segmented legs and actuator limitations (Fig 1). Our goal here is to find the simplest model of jumping that generates human-like ground forces with parameters that do not depend on jump depth.

METHODS

We modeled the physiological characteristics (Fig 1) of the actuator such that: 1) as the actuator lengthens, the force-length relationship can be constant (i.e. not limiting), linearly decreasing, or parabolic; 2) when moving, the force-velocity relationship can be constant or linearly decreasing; and 3) force activation dynamics that can be instantaneous or laggy. Using all combinations of these physiological parameters, we tested 12 different linear actuator models. To evaluate model performance and find the best fitting parameters, we collected ground reaction forces from 10 human subjects jumping from a wide range of initial starting jump depths (30 jumps each).

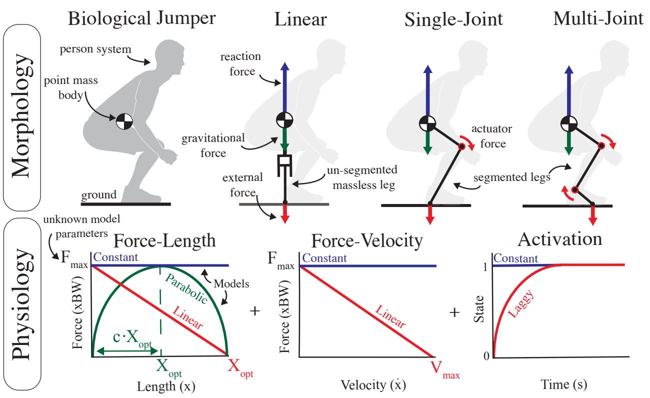

Figure 1: The morphology and physiology we tested to determine the simplest model that describes jumping behavior.

The number of unknown parameters can vary between models and at a most includes: maximum isometric force, maximum velocity, force-length parabolic width, and optimal operating length. For some models, we algebraically solve for the optimal unknown parameter. More complex models require us to solve for the best-fit unknown parameters by numerical optimization. We seek model predictions that reduce the squared error between the ground force predicted by a model and our measured data. For all solutions, we constrain the error between the predicted and measured data to be zero at jump initiation and take-off for the ground force, center of mass position, and center of mass velocity.

RESULTS AND DISCUSSION

For a linear actuator, we find that a parabolic force-length, negative linear force-velocity, and laggy activation dynamics best predicts vertical jumping behaviour (Fig 2). Other actuator physiologies in this linear morphology either do not satisfy constraints, or satisfy constraints but have poor fit. Although this simple model of the leg actuator well-predicted behavior, the optimal parameters were depth dependant. The reason for this is that for a linear actuator to meet our constraint that force is zero at the point of take-off, either the force-length or force-velocity characteristics must bring the force to zero. For most optimal solutions, we find that the width of the force-length parabola widens and contracts with depth in order for this constraint to be satisfied. We consider sensitivity to depth as evidence of an insufficient model. After all, it is the same human jumping at both shallow and deep depths. For this reason, morphology might play a role in an actuator that is depth insensitive by not requiring the physiology to satisfy constraints, a feature we will further explore as we analyze more complex morphologies (Fig 1).

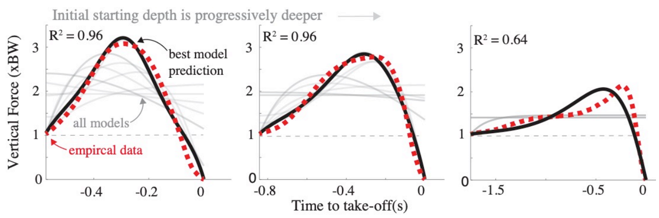

Figure 2: Representative ground reaction forces for jumps at different starting depths. Optimal solutions of 12 different models as well as the best model prediction and the empirical data shown.

CONCLUSION

With a linear actuator we find depth sensitive optimal solutions satisfying constraints imposed at jump initiation and take-off. We see this as evidence of an insufficient model.

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